

Logic circuits

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Which spell can turn on the correct indicator light in an overloaded elevator, prepare an orange soda by pressing the button on a vending machine, turn off the lights in the hallway of a house after a few minutes, or move a character on a computer screen? These and many other real-life activities are handled by logic circuits, which we will explore in more detail in the following series of problems.

Logic circuits consist of so-called logic gates that implement logic operations. We will work only with three basic logic gates, namely NOT (negation), AND (conjunction) and OR (disjunction), in the exercises. The figure shows their respective symbols (according to the American ANSI/MIL standard) in logic circuits. They are oriented so that the input direction is from the left. Inputs are understood as statements, while outputs are compound statements.

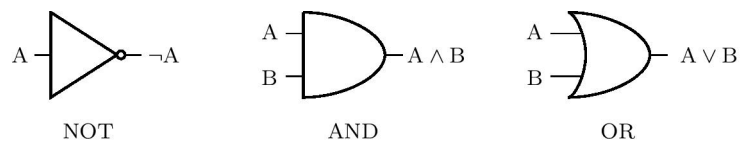


Figure 1: Logic gate symbols

The truth values are implemented in logic circuits by voltage. Low voltage indicates a truth value of 0, while a high voltage level indicates a value of 1. For example, if the AND gate has a low voltage level at input A and a high voltage level at input B, the output is a low voltage level. The specific values of the levels vary according to the specific use of the circuit. A low level of approximately 0 V and a high level of approximately 5 V are common.

In the next figure we see a representation of a more complex logic circuit. For clarity, the figure also shows the sequential compounding of statements, which correspond to the inputs or outputs of the individual gates. The black dot indicates the node at which the logic circuit branches. Thus, the output of one gate can be fed to multiple inputs at the same time.

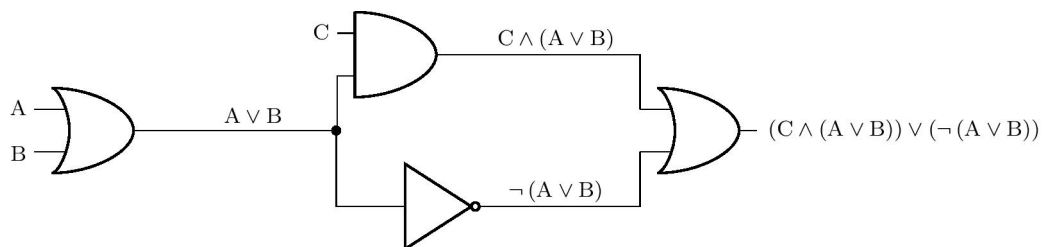


Figure 2: An example of a logic circuit

In the following exercises, switches or buttons can be placed in front of the inputs, and light bulbs can be placed behind the outputs of the logic circuit. Let's agree that the logic value at the input is equal to 1 if and only if the switch is on or the button is pressed. Similarly, a light bulb lights if and only if there is a logic value of 1 at the corresponding output.

Exercise 1. In the circuit in the previous figure, there are switches in front of inputs A, B and C, and a light bulb is connected to the output. If switch C is not on, in what position must switches A and B be in order for the bulb to light?

Exercise 2. The logic circuit shown in the figure below is given with switches at inputs A, B and C and a light bulb at output Z. Which switches must be turned on for the bulb to light? Find all solutions to the problem. If the wires cross in the diagram without a node shown, it is assumed that there is no actual contact between the wires.

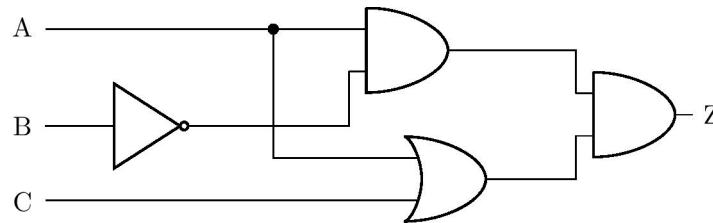


Figure 3: Assignment of exercise 2

Exercise 3. Design a logic circuit that, in the event of a failure of one of the two water pumps (or both), lights up a warning lamp at the output of the circuit. As long as the pump is working, it sends a signal corresponding to a logic one to one of the two inputs of the circuit.

Exercise 4. Modify the warning device from the previous exercise. Red and green lights will now be connected to the two outputs. If both pumps work, the green light is on and the red light is off. If one of the pumps fails, the red light also lights up, and if both pumps fail, only the red light lights up. Design the corresponding logic circuit.

Exercise 5. Design a two-input, one-output logic circuit that performs logical equivalence.

Exercise 6. After pressing the appropriate button, the coffee machine can prepare three types of drinks: lungo, macchiato and cocoa. Drinks are prepared by mixing four ingredients (hot water, milk, coffee and cocoa concentrate). Each ingredient has its own nozzle. Design a logic circuit with three inputs (one for each drink) and four outputs (one for each nozzle valve) given that lungo is prepared from water and coffee concentrate, macchiato from water, milk and coffee concentrate, and cocoa from water and cocoa concentrate.

For the sake of simplicity, let's assume that no one thinks of pressing multiple buttons at the same time, so you don't need to deal with these cases. The ingredient is released into the cup exactly when there is a logic one at the corresponding output.

All the mentioned exercises can be illustrated on various logic circuit simulators, e.g. CircuitVerse online simulator. In the last figure, the circuit from Exercise 2 is modeled using this simulator. It is also possible to use specialized electronic kits for illustration.

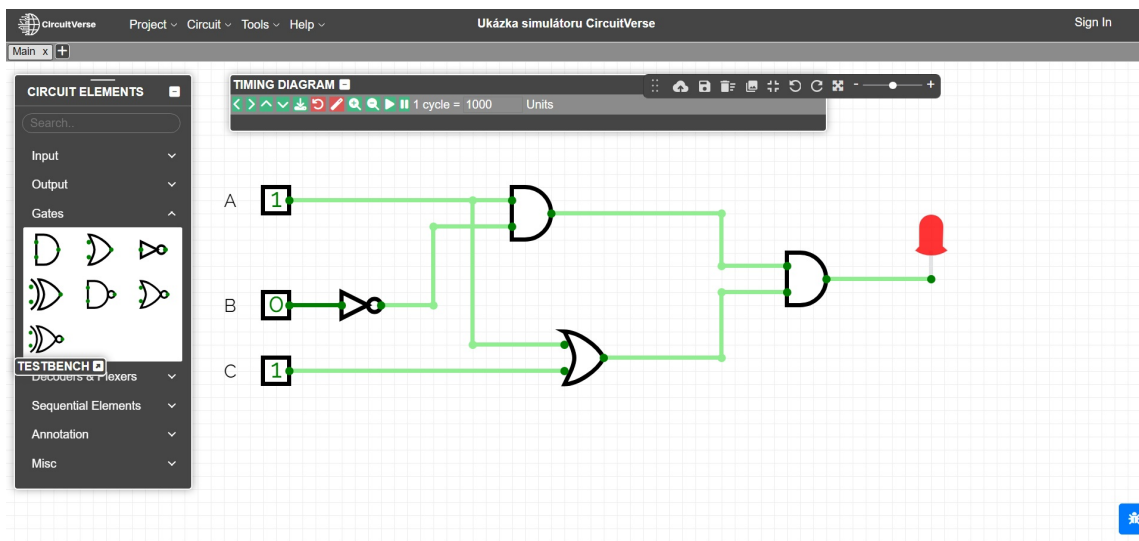


Figure 4: CircuitVerse online simulator environment

Literature

- Perrin J. P., Denouette M., Daclin E. *Logické systémy, díl I. Kombinační logické obvody. Úvod do sekvenčních obvodů*. Praha: SNTL. 1972
- *Online simulátor CircuitVerse*, <https://circuitverse.org/simulator>