

Parabolic antenna

Keywords: analytic geometry, conic sections, the equation of a parabola

A quality antenna is one of the most important elements of any radar. One type of antenna is the so-called parabolic antenna. This antenna has the shape of a rotating paraboloid (that means it was created by rotating a part of the parabola around its axis) and is used not only for radiolocation but also, for example, for receiving satellite television broadcasts.

The parabolic shape of the receiving antenna ensures that the incoming signals from a certain direction are concentrated in one point after reflection from the antenna. Namely in the focal point of the parabola, the rotation of which created the antenna. The antenna's receiver is therefore located at this point. On the other hand, if the output of a high-frequency generator (emitter) is located at the focal point, the antenna acts as a transmitter. After reflection from the paraboloid, the energy from the emitter is concentrated into a narrow stream of microwaves with parallel beams.

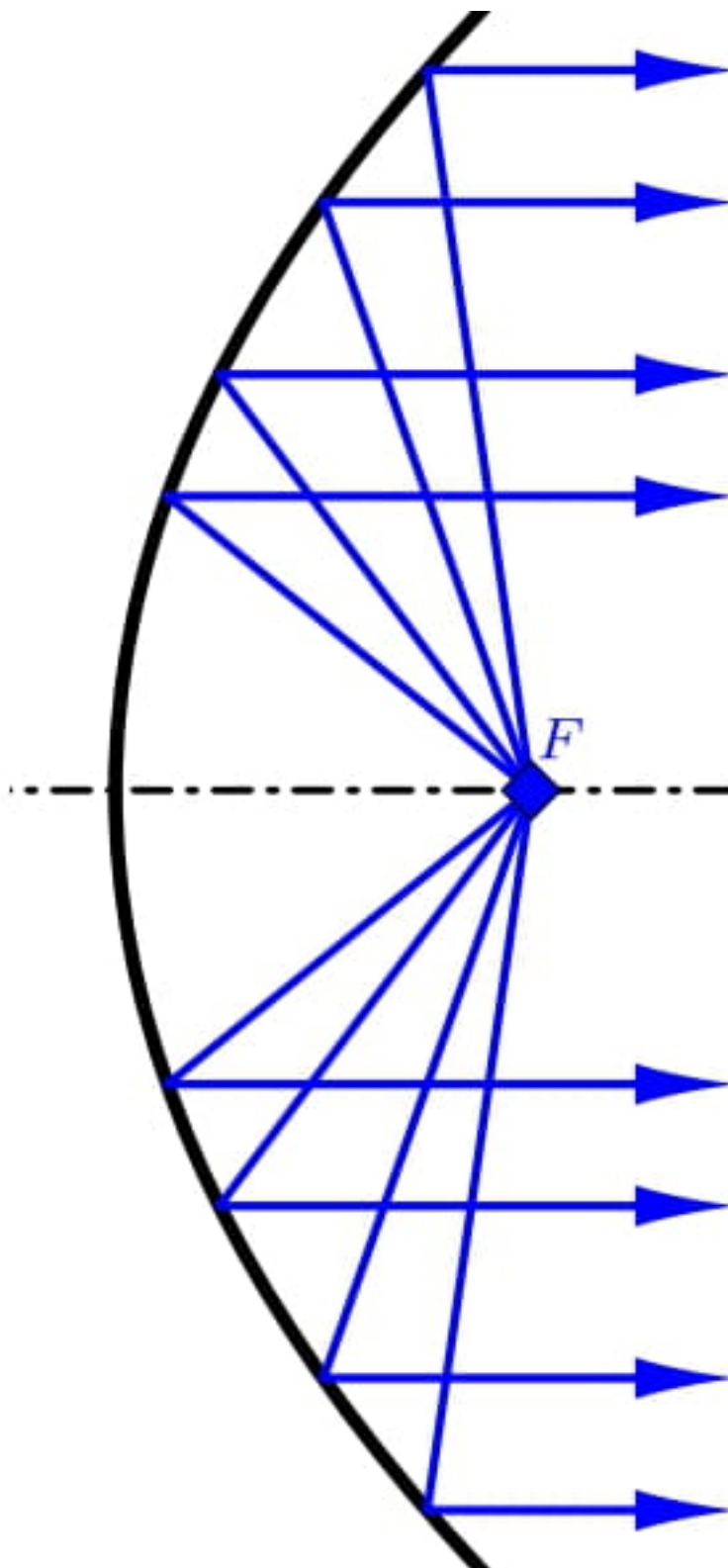


Figure 1: Section through a parabolic antenna

The axial section of the antenna is a part of the parabola. It is characterized by two parameters. They are the diameter d of the antenna and the depth h of the antenna, see the figure. These two parameters determine the position of the focus F . We denote the distance of the focus from the vertex of the parabola by f . The last important characteristic of the antenna is its opening angle, which expresses the angle at which the edges of the part of the parabola are visible from the focal point. We denote its value by 2φ .

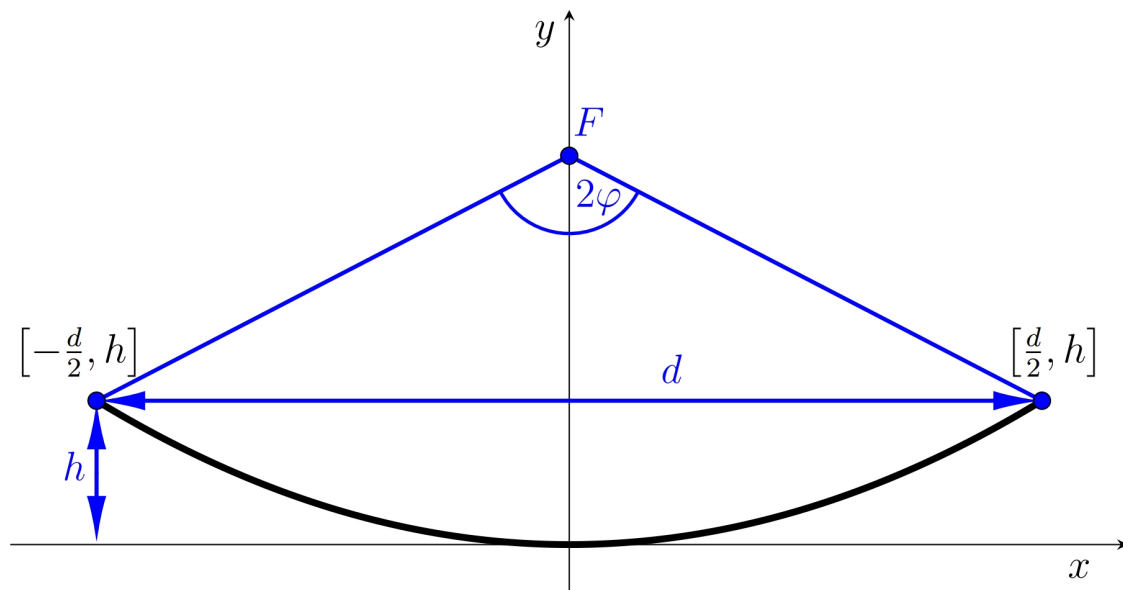


Figure 2: Parabolic antenna model

When calculating the position of the focus point, it is advantageous to assume that the location of the vertex of the parabola is at the origin of the coordinate system, and the x axis is the tangent to the parabola at its vertex. The standard equation of the parabola is then

$$x^2 = 2py,$$

where p is a parameter of the parabola (semi-latus rectum), i.e. the distance of the focus from the directrix of the parabola. For the parameter p , $p = 2f$ holds.

Assignment

Exercise 1. We have a parabolic antenna with a diameter of $d = 120$ cm and a depth of $h = 20.3$ cm. Such an antenna is suitable for use in amateur radio band 5.76 GHz (wavelength 5.2 cm). Determine the optimal location for the receiver.

Solution. For the antenna to function optimally, the receiver must be located at the focal point of the parabolic antenna. Therefore, we have to determine the coordinates of the focal point. Given the placement of the parabola, its endpoints have coordinates $[-\frac{d}{2}, h]$ and $[\frac{d}{2}, h]$. Moreover, both of these points must satisfy the standard equation of the parabola $x^2 = 2py$, where the parameter p determines the position of the focus, while $f = \frac{p}{2}$ applies in our notation.

Results matter!

Let's substitute the right endpoint into this equation and determine p :

$$60^2 = 2p \cdot 20.3.$$

Hence $p \doteq 88.7$ cm. The focus lies on the y -axis at a distance $f = \frac{p}{2} \doteq 44.3$ cm from the vertex of the parabola.

Exercise 2. Determine the quadratic function (in explicit form) expressing the curvature of a parabolic antenna and graph it (e.g. in GeoGebra).

Solution. From the standard equation of the parabola $x^2 = 2 \cdot 88.7y$ it is necessary to express the y coordinate. For the quadratic function applies

$$y = \frac{1}{177.4}x^2.$$

The graph of the quadratic function correctly captures the curvature of the parabolic antenna if both axes have the same scale.

Exercise 3. Calculate the opening angle 2φ of the parabola.

Solution. To calculate the opening of the parabola, we use a right triangle with legs of lengths $f - h$ and $\frac{d}{2}$.

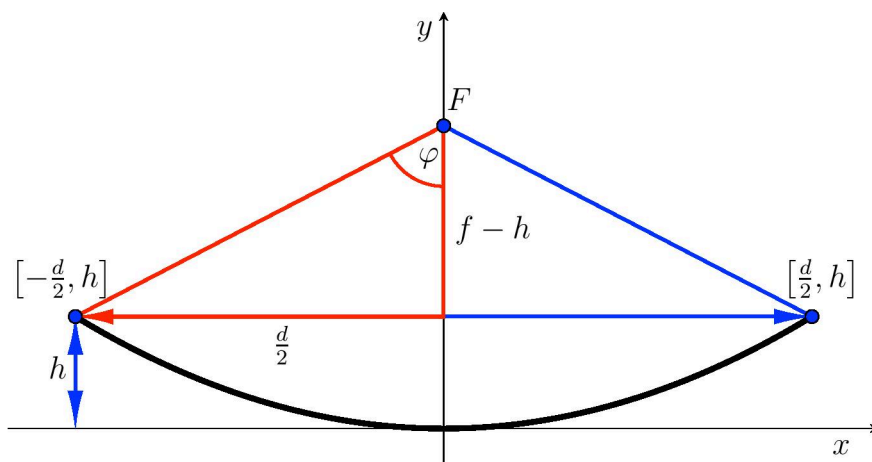


Figure 3: Triangle for calculating the opening angle

For half the opening angle we get

$$\operatorname{tg} \varphi = \frac{\frac{d}{2}}{f - h} = \frac{60}{44.3 - 20.3} \implies \varphi \doteq 68.2^\circ.$$

The opening angle of the parabola 2φ is 136.4° .

Literature

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Results matter!

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